DECENTRALIZED OBSERVERS FOR LARGE SCALE SYSTEMS

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Abstract

A new technique for decentralized state estimation in large scale control systems is proposed. Given that a large scale dynamical system can be decomposed into a number of interconnected input-output decentralized subsystems, a method for designing totally decentralized local reduced order estimators is proposed. Here the interconnection effect among the subsystems is treated as totally unknown inputs to each subsystem. The method builds upon a novel approach in designing reduced order unknown input observers (UIO) for dynamical systems, and it therefore results in totally decentralized estimators' structure. Necessary and sufficient conditions for designing such estimators are stated. Finally, the design approach is verified by a numerical example.

1) Introduction

The concept of decentralization has been of interest in systems and control due to the complexity and dimensionality of many large scale dynamical systems. Mathematical models of such complexity can be found in various fields such as engineering, economic, and ecological systems. As a result, decentralized stabilization and control of large scale systems have been studied extensively for more than two decades (e.g. Siljak (1978) and Jamshidi (1983) and references cited). In spite of this fact, problem of decentralized state estimation has generally been overlooked. Majority of the works on decentralized control are based on the assumption that the state variables are available at least locally at the subsystem level for measurement and control purposes. However, this is often not true in many physical systems due to economics, physical, or technological reasons. Therefore, decentralized state estimation is a problem of equal importance to that of control for large scale systems due to the fact that many of the work on control of such systems require the availability of state variables of the subsystems. Also, decentralized estimation is attractive since a dynamical observer of large order is impractical for a system with a large number of state variables.

A number of authors have addressed the problem of decentralized state estimation in recent years. In Siljak (1978), Siljak and Vukcevic (1976,1978), and Sundareshan (1977) the local estimators are required to communicate with one another. That is, in the dynamical model of the local estimators there is an interconnection term accounting for the interaction among the subsystems. This scheme therefore adds to the structural, and also computational complexity of the estimation problem. Ozguner (1977) addresses the problem of designing observers for a class of multilevel hierarchical systems with two time scale property. Viswanadham and Ramakrishna (1982), and Ficklscherer and Muller (1984) use the unknown input observer (UIO) theory to deal with the interconnection effect in designing local estimators, whereas Ikeda and Willems (1986) employ the so called almost disturbance decoupling concept to design decentralized observers and to deal with the effect of the interactions among the subsystems.

In this paper we propose a totally decentralized observer design scheme for estimating the states of a decentralized large scale system. It is well known that if standard observer

theory is used to design local state estimators for local subsystems without any regard for the interaction terms, then unsatisfactory performance of the system can be expected when the local observers are used in the composite system. The reason for this is that it can easily be shown that the estimation error for each subsystem will be governed by the unaccounted interconnection term and there will in general be no guarantee for the convergence of the estimate. It is therefore necessary to account for the interaction terms in designing the decentralized observer. In this paper, as in Viswanadham and Ramakrishna (1982), and Ficklscherer and Muller (1984), the interconnections terms from other subsystems are treated as unknown inputs at each of the local subsystems. In this way they will be accounted for when designing the local observers, however, since they are assumed to be unknown, there is no need for communication among the local observers as was the case in Siljak and Vukcevic and (1976,1978), and Sundareshan (1977). This in turn will result in a great deal of simplicity and reduction of data transfer and computation at the local subsystems.

Different algorithms for design of the UIO have been discussed in Viswanadham and Ramakrishna (1982), Kobayashi, and Nakamizo (1982), Kudva et. al. (1980), Miller and Mukundan (1982), Kurek (1983), and Yang and Wilde (1988). In this paper we present a new approach to designing UIOs with applications to decentralized state estimation. In addition necessary and sufficient conditions for existence of such observers are given. It is believed that the complexity and amount of computation in designing decentralized state estimators have been greatly reduced in the proposed approach when compared to the aforementioned approaches.

2) Statement of the Problem

Consider a large scale linear time invariant dynamical system represented in the following state space formulation

$$S: \quad \dot{x} = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

where $x \in \Re^n$, $u \in \Re^r$, and $y \in \Re^p$ are the state, input, and the output of the system, respectively. Assume that the large scale system S has been decomposed into r coupled subsystems, the ith of which (S_i) represented by the following input/output decentralized state space representation

$$S_i: \qquad \dot{x}_i = A_i x_i + B_i u_i + \sum_{\substack{j=1 \\ j \neq i}}^{r} A_{ij} x_j \tag{3}$$

$$y_i = C_i x_i \tag{4}$$

where $x_i \in \Re^{n_i}$, $u_i \in \Re^{q_i}$, $y_i \in \Re^{p_i}$, are the state, control, and output vectors of the ith subsystem. In addition matrices A_i, A_{ij}, B_i, C_i are all constant matrices of appropriate dimension. It is clear that the subsystem S_i interacts with the others through the terms containing A_{ij}.

Assume that the triplet (A_i, B_i, C_i) is a minimum realization, and the rank of $C_i = p_i$ for all i=1,2,...,r. In addition, since we are dealing with the estimation problem, it is assumed that there exists a decentralized feedback law of the form

$$u_i = u_{il} + u_{ic} = K_i x_i + u_{ic} \tag{5}$$

which would stabilize the composite system S. In (5), u_{il} is a local state feedback type of control law obtained for the ith subsystem without any regard to the interconnection effects, and u_{ic} is the global or corrective control law applied at the ith subsystem to counteract the effect of the interactions among the subsystems.

It is clear from that the control law of the form (5) requires the availability of the state variables of the local subsystems for measurement and feedback. Assuming that not all of the local state variables are measurable for each subsystem, the problem of interest is then to design r local observers to estimate the unmeasurable components of x_i, i=1,2,...,r. The structural constraint on the desired estimators is that no exchange of information among these local estimators are allowed. That is, the ith estimator is to utilize only the local input and output measurements to estimate the entire state vector x_i. The structure, necessary and sufficient conditions for existence, along with a design procedure for implementing such estimators is presented for the ith subsystem in the following discussion.

3) Design of the Decentralized Estimators

Let us rewrite the equations (2), and (3) in a form of local input/output terms as[1]

$$\dot{x}_i = A_i x_i + B_i u_i + D_i v_i \tag{6}$$

$$y_i = C_i x_i = [0 \qquad I] x_i \tag{7}$$

where

$$v_i = R_i \bar{v}_i \tag{8}$$

$$\bar{D}_i = D_i R_i \tag{9}$$

$$\bar{D}_i = [A_{i1}, A_{i2}, \ldots, A_{i(i-1)}, A_{i(i+1)}, \ldots, A_{is}] \qquad with \quad \rho(\bar{D}_i) = m_i \tag{10}$$

$$\bar{v}_i = \begin{pmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_{i-1} \\ x_{i+1} \\ \cdot \\ \cdot \\ x_s \end{pmatrix} \tag{11}$$

where $D_i \in \Re^{n_i \times m_i}$ with $\rho(D_i) = m_i$, and $v_i \in \Re^{m_i}$. Notice that without any loss of generality a special structure for C_i's are assumed. This is not a restrictive assumption since as long as C_i's are of full rank, there will always exit a similarity transformation that can bring them into the desired form given in (7) (Chen (1985)).

Let us consider designing an estimator for the ith subsystem. Since the estimator structure is constrained to utilize only local information, the interconnection effects (i.e. the last term in (7)) will be treated as a completely unknown input to the system. In order to estimate x_i of each subsystem S_i, an estimator of the following form is proposed

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1 From here on I will denote an identity matrix of appropriate dimension.

$$\dot{w}_i = F_i w_i + E_i y_i + L_i u_i \tag{12}$$

with

$$\hat{x}_i = \overline{M}_i w_i + \overline{N}_i y_i \tag{13}$$

where $w_i, F_i, E_i, L_i, \overline{M}_i$, and $\overline{N}_i$ are yet to be given.

There are now two separate cases that needs to be addressed in designing the local estimators:

Case I - $p_i \geq m_i$

Given the observer structure in (12), and (13), necessary condition for the existence of such observer is stated in the following.

Theorem 1- The unmeasurable state variables of the ith subsystem given in (6-7) can be estimated using an observer of the form (12-13) if the following necessary condition is satisfied:

$$\rho\begin{pmatrix} D_{i2} \\ D_{i3} \end{pmatrix} = m_i$$

where

$$D_i = \begin{pmatrix} D_{i1} \\ D_{i2} \\ D_{i3} \end{pmatrix} \tag{14}$$

and $D_{i1} \in \Re^{(n_i - p_i)\times m_i}$, $D_{i2} \in \Re^{(p_i - m_i)\times m_i}$, $D_{i3} \in \Re^{m_i \times m_i}$..

Proof: see Appendix A.

Let us rewrite equations (6), and (7) in the following partitioned form

$$\dot{x}_i = \begin{pmatrix} A_{i1} \\ A_{i2} \\ A_{i3} \end{pmatrix} x_i + \begin{pmatrix} B_{i1} \\ B_{i2} \\ B_{i3} \end{pmatrix} u_i + \begin{pmatrix} D_{i1} \\ D_{i2} \\ D_{i3} \end{pmatrix} v_i \tag{15}$$

$$\begin{pmatrix} y_{i1} \\ y_{i2} \end{pmatrix} = \begin{pmatrix} 0 & I & 0 \\ 0 & 0 & I \end{pmatrix} x_i \tag{16}$$

where the matrices A_{ik}, B_{ik}, and D_{ik} (k=1,2,3) all have appropriate dimensions, and the state vector x_i is partitioned as

$$x_i = \begin{pmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \end{pmatrix} = \begin{pmatrix} x_{i1} \\ y_{i1} \\ y_{i2} \end{pmatrix}$$

and $x_{i1} \in \Re^{n_i - p_i}$ is the vector whose estimate is required.

Theorem 2 - The state of the subsystem S_i given in (6-7) can be estimated using the local estimator described (12-13) if and only if the pair $(\overline{A}_{i21}, \overline{A}_{i11})$ is observable. In addition, under such conditions the eigenspectrum of F_i can be assigned arbitrarily, and the variables in (12) and (13) are defined as

$$H_i = (\overline{A}_{i13} - M_i\overline{A}_{i23}) + (\overline{A}_{i11} - M_i\overline{A}_{i21})(D_{i1} - M_iD_{i2})D_{i3}^{-1} \tag{17}$$

$$G_i = (\overline{A}_{i12} - M_i\overline{A}_{i22}) + (\overline{A}_{i11} - M_i\overline{A}_{i21})M_i \tag{18}$$

$$F_i = (\overline{A}_{i11} - M_i\overline{A}_{i21}) \tag{19}$$

$$L_i = (\overline{B}_{i1} - M_i\overline{B}_{i2}) \tag{20}$$

$$E_i = [G_i \quad H_i] \tag{21}$$

$$\overline{A}_{ik} = A_{ik} - D_{ik}D_{i3}^{-1}A_{i3} \quad \textit{for} \quad k = 1,2 \tag{22}$$

$$\overline{B}_{ik} = B_{ik} - D_{ik}D_{i3}^{-1}B_{i3} \quad \textit{for} \quad k = 1,2 \tag{23}$$

$$\overline{A}_{ik} = [\overline{A}_{ik1} \quad \overline{A}_{ik2} \quad \overline{A}_{ik3}] \quad \textit{for} \quad k = 1,2 \tag{24}$$

$$\overline{M}_i = \begin{pmatrix} I \\ 0 \end{pmatrix} \quad \overline{N}_i = \begin{pmatrix} N_i \\ I \end{pmatrix} \tag{25}$$

$$N_i = [M_i \quad (D_{i1} - M_iD_{i2})D_{i3}^{-1}] \tag{26}$$

$$w_i = \hat{x}_{i1} - [(D_{i1} - M_iD_{i2})D_{i3}^{-1}y_{i2} + M_iy_{i1}] \tag{27}$$

Proof: See Appendix A.

Case II - $p_i = m_i$

Let us rewrite equation (6) in the following partitioned form

$$\begin{pmatrix} \dot{x}_{i1} \\ \dot{y}_i \end{pmatrix} = \begin{pmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{pmatrix}\begin{pmatrix} x_{i1} \\ y_i \end{pmatrix} + \begin{pmatrix} B_{i1} \\ B_{i2} \end{pmatrix}u_i + \begin{pmatrix} D_{i1} \\ D_{i2} \end{pmatrix}v_i \tag{28}$$

Note that in the above representation D_{i2} is a full rank square matrix, and is therefore invertible.

Theorem 3 - The eigenspectrum of the decentralized UIOs can not be arbitrarily assigned if $p_i = m_i$. In this case, an asymptotically stable observer with fixed eigenspectrum of the form (12) would exist iff the following matrix is negative definite.

$$\mathbf{F}_i = (\mathbf{A}_{i11} - \mathbf{D}_{i1}\mathbf{D}_{i2}^{-1}\mathbf{A}_{i21}) \tag{29}$$

where the state estimate is given by

$$\hat{\mathbf{z}}_{i1} = \mathbf{w}_i + \mathbf{M}_i\mathbf{y}_i \tag{30}$$

and

$$\mathbf{L}_i = \mathbf{B}_{i1} - \mathbf{D}_{i1}\mathbf{D}_{i2}^{-1}\mathbf{B}_{i2} \tag{31}$$

$$\mathbf{E}_i = \Sigma_i\mathbf{A}_i\Gamma_i \tag{32}$$

where

$$\Sigma_i = [\mathbf{I} \quad -\mathbf{D}_{i1}\mathbf{D}_{i2}^{-1}] \qquad \Gamma_i = \begin{pmatrix} \mathbf{D}_{i1}\mathbf{D}_{i2}^{-1} \\ \mathbf{I} \end{pmatrix}$$

Proof: See Appendix A.

4) Illustrative Example

To illustrate the applicability of the decentralized estimation scheme presented here, consider the following MIMO system

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -2 & -1 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ -8 & 1 & -1 & -1 & -2 & 0 \\ 4 & -0.5 & 0.5 & 0 & 0 & -4 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}\mathbf{x}$$

Using the following to decompose the above large scale system

$$\mathbf{D}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \mathbf{R}_1 = [-1 \quad 1 \quad 1]$$

$$\mathbf{D}_2 = \begin{pmatrix} 0 \\ 1 \\ -0.5 \end{pmatrix} \qquad \mathbf{R}_2 = [-8 \quad 1 \quad -1]$$

will result in the following two interconnected subsystems, where the interconnections are now being treated as unknown inputs to each subsystems (i.e. v_1, and v_2)

$$S_1: \quad \dot{\mathbf{x}}_1 = \begin{bmatrix} -1 & 0 & 0 \\ -1 & 0 & 1 \\ 1 & -2 & -1 \end{bmatrix} \mathbf{x}_1 + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \mathbf{u}_1 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{v}_1$$

$$\mathbf{y}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x}_1$$

$$S_2: \quad \dot{\mathbf{x}}_2 = \begin{bmatrix} -1 & 0 & 0 \\ -1 & -2 & 0 \\ 0 & 0 & -4 \end{bmatrix} \mathbf{x}_2 + \begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{u}_2 + \begin{bmatrix} 0 \\ 1 \\ -0.5 \end{bmatrix} \mathbf{v}_2$$

$$\mathbf{y}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x}_2$$

It should be pointed out that the open loop systems S_1, and S_2 are stable with their eigenspectrum located at {-0.5±j1.3299,-1} and {-1,-2,-4} respectively. Also, note that $p_i > m_i$ for i=1,2.

From the above it is clear that at each subsystem one of the state variables has to be estimated, and therefore two local first order observers are needed. It was also decided that observers' pole be placed at -4, and -16 for the S_1, and S_2 respectively. From (A6) we obtain

$$\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{T}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

The following state estimators can be obtained by using (A5),(12),(13), and (17-27)

$$S_1: \quad \dot{w}_1 = -4w_1 + [12 \quad 3]y_1 + [3 \quad 1]u_1$$

$$\hat{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} w_1 + \begin{bmatrix} -3 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} y_1$$

$$S_2: \quad \dot{w}_2 = -16w_2 + [210 \quad 360]y_2 + [15 \quad 29]u_2$$

$$\hat{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} w_2 + \begin{bmatrix} -15 & -30 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} y_2$$

For the purpose of illustrating the decentralized estimation approach discussed here, the performance of the estimators for an initial disturbance of $x_1(0) = [-2 \quad 2 \quad 0]^T$, and $x_2(0) = [3 \quad 1 \quad 0]^T$ was considered. Figures one and two show the actual unmeasurable state variables (solid lines) along with their estimate (dotted lines) for each of the two subsystems for a unit step input at each control channel. From these figures it is clear that the decentralized estimators perform satisfactorily, as expected.

5) Conclusions

A new design approach for decentralized state estimation in large scale systems was proposed in this paper. Due to computational or geographical reasons the large scale dynamical system is often broken up in a number of input/output decentralized interconnected systems of lower order than the overall composite system for stabilization and control purposes. It was shown here that by treating the interconnection terms as unknown inputs to each subsystems, it may be possible to design totally decentralized reduced order local observers for estimating the unmeasurable state variables of each subsystem. It was shown that the method fails if the number of unknown inputs are more than the outputs at each subsystem. Also, it is possible to assign the eigenspectrum of the estimators arbitrarily if the number of unknown inputs are less than the measured outputs. This is not true when the number of unknown inputs are equal to the outputs. On the other hand, the fact that the estimators do not require any exchange of information among themselves greatly reduces the computational requirement, and perhaps adds to the overall reliability of the system. A new approach for designing such local estimators was presented which is believed to be simpler than previously proposed schemes.

Figure 1 - Actual and estimated trajectories for subsystem one

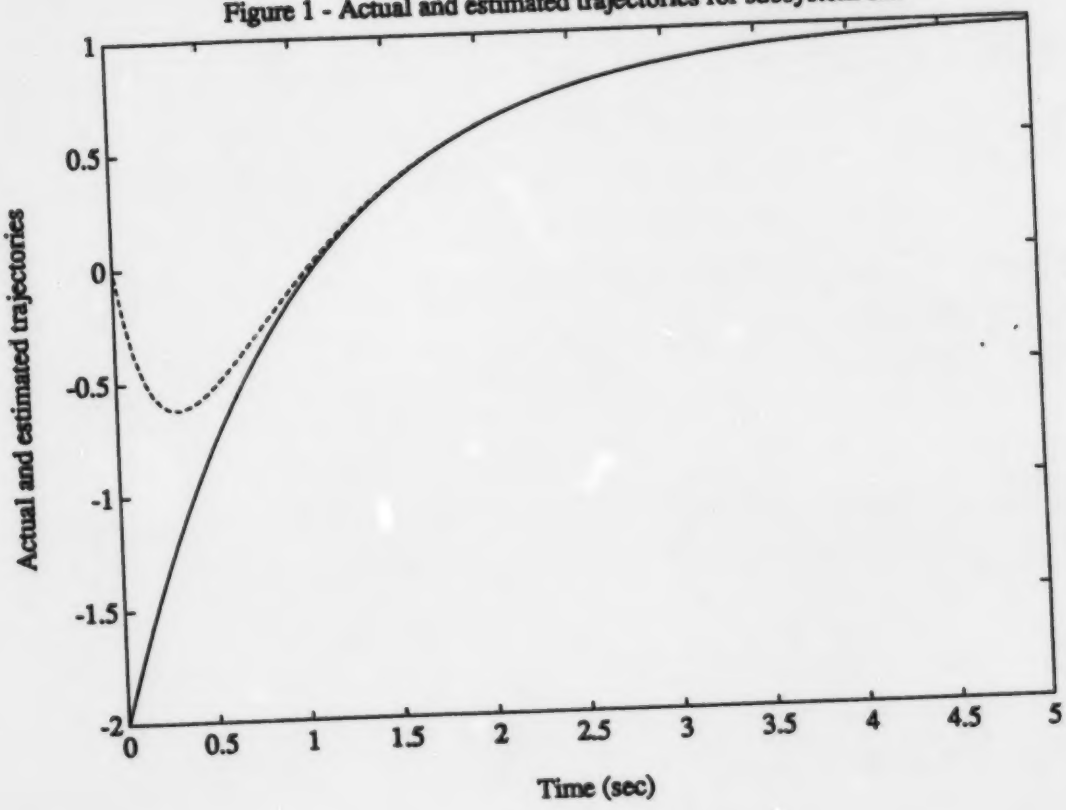
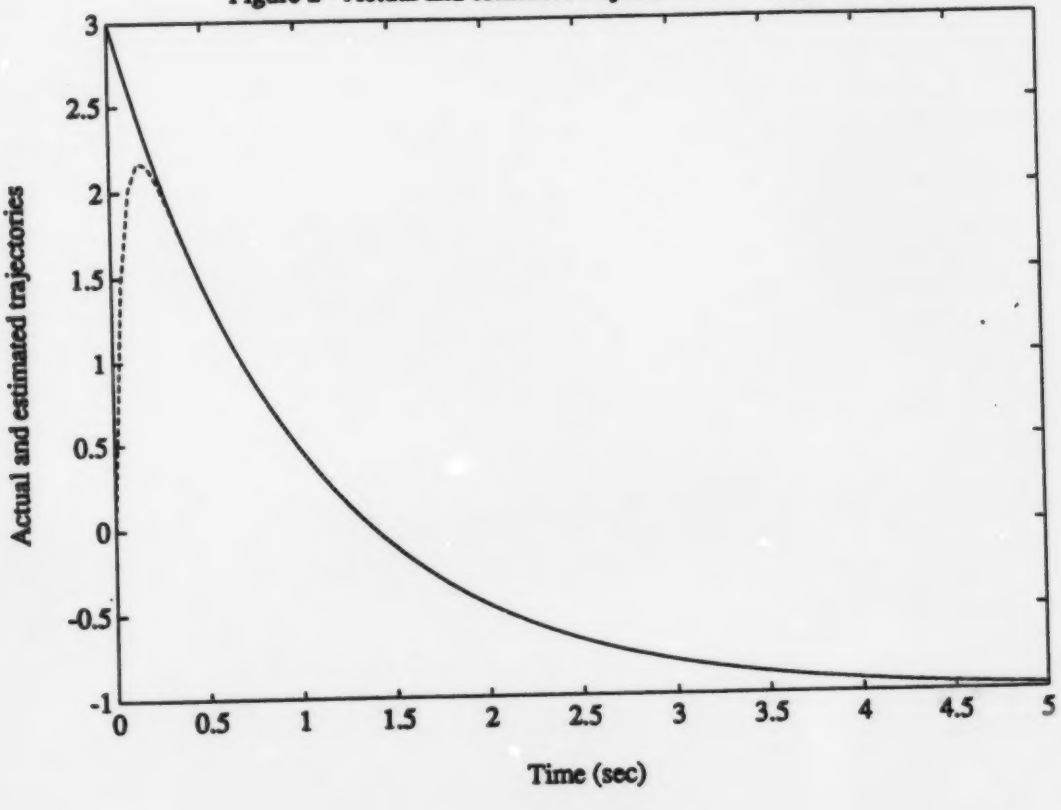


Figure 2 - Actual and estimated trajectories for subsystem two



In summary, the totally decentralised estimation approach proposed here, as well as those similar schemes proposed by Ficklscherer and Muller (1984), and Viswanadaham and Ramakrishna (1982) impose some restriction on the large scale system structure, specifically the interconnection pattern, and therefore, may not be applicable in some situations. However, when applicable, it leads to a very simple and appealing alternative for decentralized estimation.

Appendix A

Proof of theorem 1

Given the state estimator as in (12), and (13), define the estimation error as

$$e_i = x_i - \hat{x}_i \tag{A1}$$

Suppose a stable observer exist for estimating the state vector, then the error dynamics should be asymptotically stable independent of the inputs and outputs. That is, there must exist a negative definite matrix Q_i such that

$$\dot{e}_i = Q_i e_i \tag{A2}$$

Combining (12),(13),(A1),(A2), and using (14) will result in

$$(P_iA_i - Q_iP_i - \overline{M}_iE_iC_i)x_i + (P_iB_i - \overline{M}_iL_i)u_i + P_iD_iv_i + (Q_i\overline{M}_i - \overline{M}_iF_i)w_i = 0 \tag{A3}$$

where

$$P_i = I - \overline{N}_iC_i$$

Equation (A3) holds iff the following are satisfied

$$P_iA_i - Q_iP_i - \overline{M}_iE_iC_i = 0$$

$$P_iB_i - \overline{M}_iL_i = 0$$

$$Q_i\overline{M}_i - \overline{M}_iF_i = 0$$

$$P_iD_i = 0$$

The last equation in the above implies that

$$\overline{N}_i\begin{pmatrix} D_{i2} \\ D_{i3} \end{pmatrix} = D_i$$

Since D_i is of full rank, there exists a solution to $\overline{N}_i$ iff

$$\rho\begin{pmatrix} D_{i2} \\ D_{i3} \end{pmatrix} = m_i \qquad \textit{Q.E.D}$$

Remark 1 - Notice that the above theorem and proof implies that $p_i \geq m_i$.

Proof of Theorem 2

From the necessary condition of Theorem 1, without any loss of generality we can assume that D_{f3} is invertible (if it is not relabel the equation so that it becomes invertible). In that case post multiplying (15) by T_f would yield

$$\begin{pmatrix} \dot{x}_{f1} - D_{f1}D_{f3}^{-1}\dot{y}_{f2} \\ \dot{y}_{f1} - D_{f2}D_{f3}^{-1}\dot{y}_{f2} \\ \dot{y}_{f2} \end{pmatrix} = \begin{pmatrix} \overline{A}_{f1} \\ \overline{A}_{f2} \\ A_{f3} \end{pmatrix} x_f + \begin{pmatrix} \overline{B}_{f1} \\ \overline{B}_{f2} \\ B_{f3} \end{pmatrix} u_f + \begin{pmatrix} 0 \\ 0 \\ D_{f3} \end{pmatrix} v_f \tag{A5}$$

where

$$T_f = \begin{pmatrix} I & 0 & -D_{f1}D_{f3}^{-1} \\ 0 & I & -D_{f2}D_{f3}^{-1} \\ 0 & 0 & I \end{pmatrix} \tag{A6}$$

The following can now be easily obtained from the first two rows of the (A5)

$$\dot{x}_{f1} = \overline{A}_{f11}x_{f1} + r_f \tag{A7}$$

$$z_f = \overline{A}_{f21}x_{f1} \tag{A8}$$

where

$$r_f = \overline{A}_{f12}y_{f1} + \overline{A}_{f13}y_{f2} + D_{f1}D_{f3}^{-1}\dot{y}_{f2} + \overline{B}_{f1}u_f$$

$$z_f = \dot{y}_{f1} - D_{f2}D_{f3}^{-1}\dot{y}_{f2} - \overline{A}_{f22}y_{f1} - \overline{A}_{f23}y_{f2} - \overline{B}_{f2}u_f$$

The dynamical system given in (A7), and (A8) is driven by a completely known input r_f, therefore a conventional Luenberger observer of the following form can be used to estimate its state.

$$\dot{\hat{x}}_{f1} = \overline{A}_{f11}\hat{x}_{f1} + r_f + M_f(z_f - \overline{A}_{f21}\hat{x}_{f1}) \tag{A9}$$

where M_f is the estimator's gain. It is well known that the necassary and sufficient conditions for existence and arbitrary eigenspectrum assignment of such estimator is that the pair $(\overline{A}_{f21}, \overline{A}_{f11})$ be observable (Chen (1985)).

Substituting for r_i, and z_i in (A9), and eliminating the derivative of the output in the resulting equation by using (27) will result in the desired observer given in (12), and (13). *Q.E.D.*

Proof of Theorem 3:

Given the following estimation error

$$e_i = x_{i1} - \hat{x}_{i1} \qquad (A10)$$

To insure the convergence of the state estimates to the actual values, it is required that the error dynamics to be asymptotically stable independent from the inputs and outputs. Assuming that the error dynamics is governed by the following where Q_i is a negative definite matrix

$$\dot{e}_i = Q_i e_i \qquad (A11)$$

Now using (12),(28),(30),(A10), and (A11) will result in

$$(A_{i11} - M_i A_{i21} - Q_i)x_{i1} + (B_{i1} - M_i B_{i2} - L_i)u_i + (Q_i - F_i)w_i + (D_{i1} - M_i D_{i2})v_i +$$
$$(A_{i12} - M_i A_{i22} + Q_i M_i - E_i)y_i = 0 \qquad (A12)$$

Equation (A12) implies that

$$F_i = Q_i = (A_{i11} - M_i A_{i21})$$
$$L_i = (B_{i1} - M_i B_{i2})$$
$$E_i = (A_{i12} - M_i A_{i22} + Q_i M_i)$$
$$D_{i1} = M_i D_{i2}$$

Since D_{i2} is of full rank, the last of the above set of equations implies

$$M_i = D_{i1} D_{i2}^{-1}$$

Substituting M_i in the previous set of equation will prove the theorem. *Q.E.D.*

Remark 2 - Notice that if D_i was zero, then the problem would have reduced to designing a conventional Luenberger observer. That is, (assuming the observability of the system) the observer gain (M_i) could have been arbitrarily selected for assigning the eigenspectrum of the closed loop observer.

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Centre for Systems Science

• Simon Fraser University established the **Centre for Systems Science** as a provincial centre for advanced study. The objective of CSS is twofold: coordinate and combine advanced systems research at SFU, and encourage application of research results within industry in order to further the technological development of British Columbia.

•CSS has assembled three research groups to streamline its efforts. The orientation of these groups is as follows: intelligent systems, microelectronics, and computer and communications systems. Because of shared research interests, this latter group merges with LCCR (**Laboratory for Computer and Communications Research**) for the publication of technical reports.

• CSS's three groups involve over 80 skilled faculty researchers principally from SFU's Schools of Engineering Science and Computing Science, but also draw substantially on Mathematics, Physics, Geography, Philosophy, Linguistics, Psychology, Communication, and Kinesiology.